

Logic Design (Part 5)

Sequential Logic Devices & Sequential Circuits

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Combinational vs. Sequential

•Combinational Circuit

- always gives the same output for a given set of inputs
 - ex: adder always generates sum and carry, regardless of previous inputs

•Sequential Circuit

- stores information
- output depends on stored information (state) plus input
 - so a given input might produce different outputs, depending on the stored information

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Sequential Circuits – The agenda

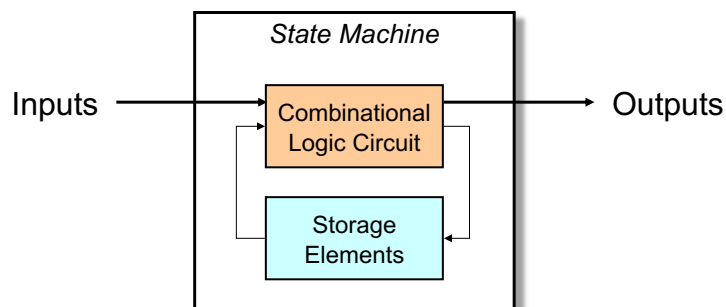
- Definition of sequential circuits
 - Components of a sequential circuit
- common storage ‘devices’ (built from latches)
 - Register
 - Memory
- synchronization using a **CLOCK**
- Modifying latches to work with a clock...Flip Flops
 - These become the basic unit of storage in sequential circuits
- Designing sequential circuits – methodology
 - Finite state machine diagrams
 - Mapping to truth table.....build circuit

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Sequential Circuit Schematic & State Machine

- sequential circuit
 - Combines combinational logic with **storage**
 - “Remembers” state, and changes output (and state) based on **inputs** and **current state**



- 1.combinational circuit to compute output and next state
- 2.storage elements to store state

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Sequential Circuits: Finite State Machines

- The behavior of sequential circuits can be expressed using characteristic tables or finite state machines (FSMs).
 - FSMs consist of a set of nodes that hold the states of the machine and a set of arcs that connect the states.
 - Directed graph to represent a FSM
- Moore and Mealy machines are two types of FSMs that are equivalent.
 - They differ only in how they express the outputs of the machine.
 - **Moore machines place outputs on each node/state**
 - Associate an output with each state
 - Mealy machines present their *outputs on the transitions*.

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FSM Design Process

- The first step is to model the behavior of the machine
 - Based on problem statement
 - Identify what the inputs are
 - Identify the outputs
 - Determine what needs to be stored to capture the “state” of the machine
- Represented as a graph – finite state diagram
 - Nodes: States – a state stores summary of events (until current time)
 - Edges: Transition from current state to next state
 - Based on input and current state
 - Computed by combinational logic
 - Outputs: Using Moore machine, determine value of outputs at each state

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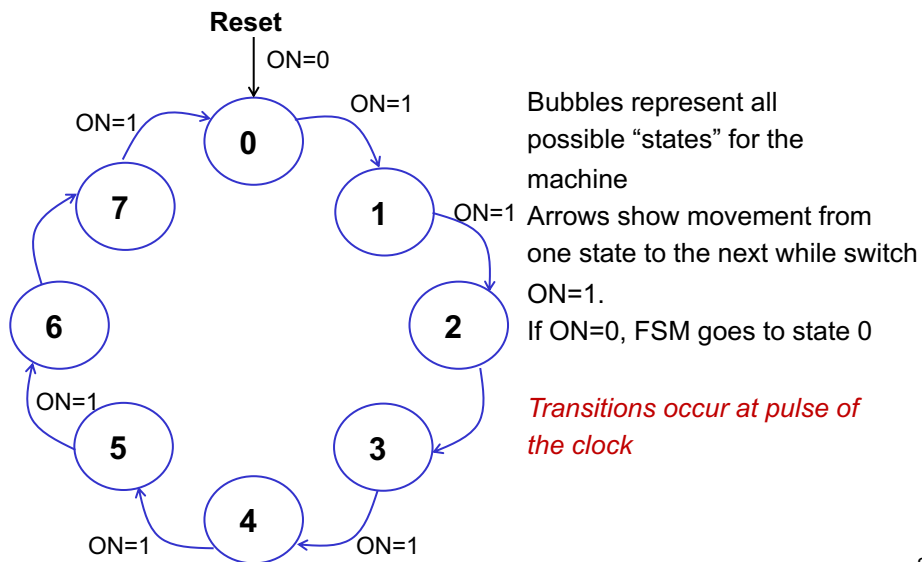
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Design a Counter: counts from 0 to 7

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Finite State Machine Representation of Counter: Counter to count from 0 to 7 (while switch is ON)



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Example: A Vending Machine

- Accept user input (coins), when total is at least 50 cents dispense output (candy)
 - We will not model the change to be returned, and only care if the input is at least 50 cents
- Input valid coins:
 - Q (25cents) D (10) or N (5)
- What should it keep track of ?
 - current total
 - Is it 50 cents or more ?
- When it reaches 50 or more:
 - Generate output
- States of the machine ?
 - What should each state capture ?
 - How many states ?

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Sequential Logic

- Where do we start:
- Build a device, using combinational logic devices, to **store** a value...*done!!*
 - D Latch (and RS Latch) – stores 1 bit
 - concept of memory
- Build other storage devices using the D Latch and logic devices we have at our disposal (i.e., in our *library*)
- What is the methodology behind design of sequential logic circuits
 - Finite State Machines to Truth Tables to Circuit
- Combine sequential and combinational logic devices to “assemble” a simple processor!

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Latches and Flip-Flops and Clock

- Latch: basic circuit for storage
 - Operate on changes in Level (i.e., 1 or 0)
 - D-Latch can store 1 bit
- Flip-flop:
 - Sequential circuits take input from output of storage
 - Latches that work on change of level can lead to unstable sequential circuits
 - As level changes the outputs change --- inputs change!
 - Flip-Flop circuits designed to operate properly when they are part of a sequential circuit
 - Modify D Latch to get a D Flip Flop (DFF)
 - Flip Flop changes state at the 'instant' that the level changes
- Clock:
 - Need to coordinate and synchronize when states change...
 - Use Clock to enable or disable the devices in a timed manner

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Our Latches

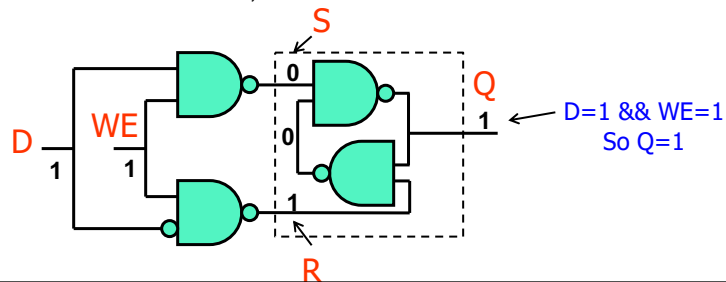
- We have designed two devices capable of storage
- RS Latch
 - Cross coupled NAND gates
 - Two inputs S (set) and R (reset) and Two outputs Q and Q'
- D Latch
 - Built from RS Latch
 - Two inputs D (value to be set), WE (write enable) and two outputs Q and Q'

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Recall the Gated D-Latch: Our 1-bit storage element

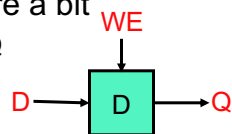
- Add logic to an R-S latch
 - Create a more convenient interface, prevent $S=0 \ \&\& \ R=0$
- Two inputs: D (data) and WE (write enable)
 - When $WE = 1$, latch is set to value of D
 - $S = \text{NOT}(D), R = D$
 - When $WE = 0$, latch continues to hold previous value
 - $S = R = 1$ (hold condition for SR latch)
- Extra logic does not allow $S=0, R=0$ case to occur



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Next... Storage Devices

- we now have a device (D-Latch) that can store a bit
 - Abstract the device: input D, WE; output/storage Q



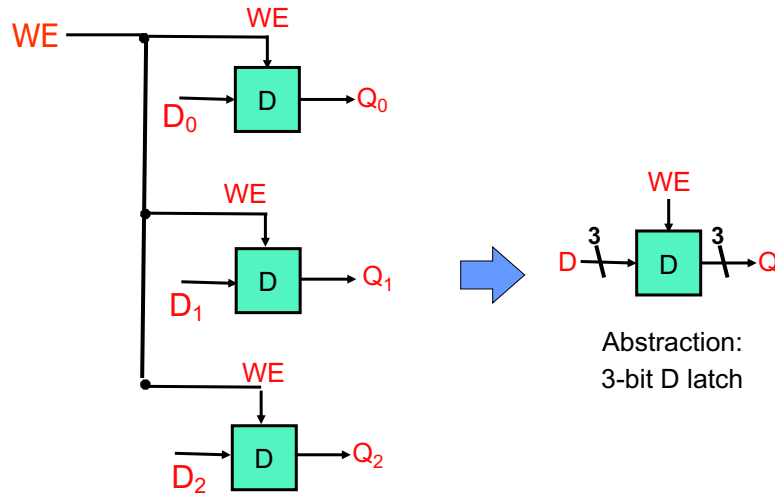
- Use this to build 'real' storage devices....
- Temporary storage in a computer...Register
 - Where are variables stored before being sent to the arithmetic unit for operations on them?
 - Can we build an n-bit register using latches?
- What about "main" memory
 - Memory hierarchy ?

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Multi-Bit D-Latch: Register ?

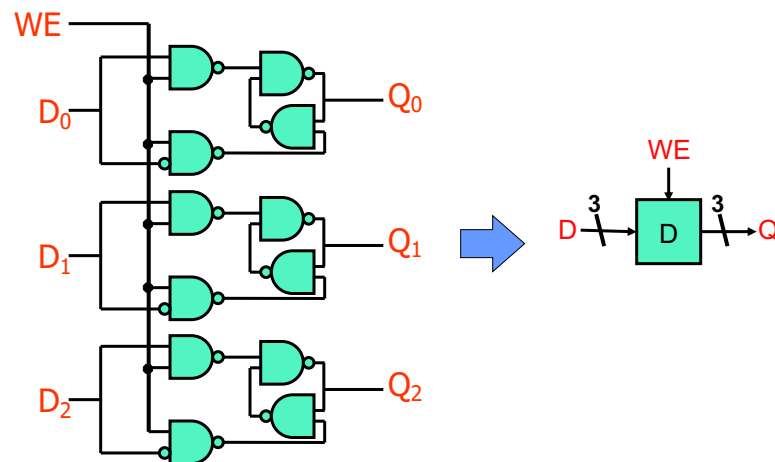
- A collection of D-latches, controlled by a common WE
- When WE=1, 3-bit value D is written to the outputs



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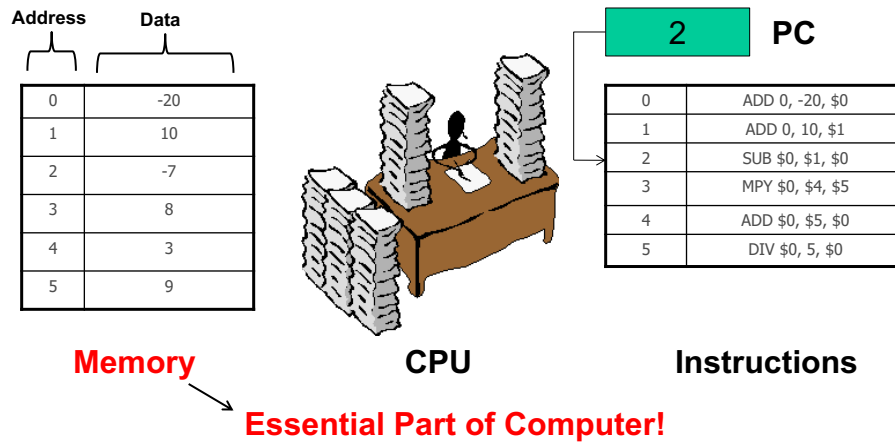
Multi-Bit D-Latch – Register: Inside the latches

- A collection of D-latches, controlled by a common WE
- When WE=1, n-bit value D is written to the outputs



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Recall: A Basic Model of a Computer



Basic Components: Address: Looks up data
Note: both are in binary

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Memory

- We know how to store m-bit number in a register
- How about many m-bit numbers?
 - Bank of registers?
- How to fetch a specific m-bit number?
 - **addressing**

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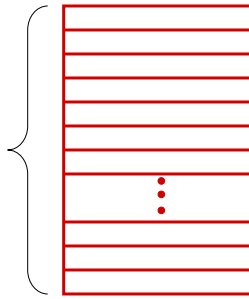
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Memory

- Now that we know how to store bits, we can build a memory – a logical k by m array of stored bits

Address Space:
number of locations
(usually a power of 2)

$k = 2^n$
locations



Addressability:
number of bits per location
(e.g., byte-addressable)

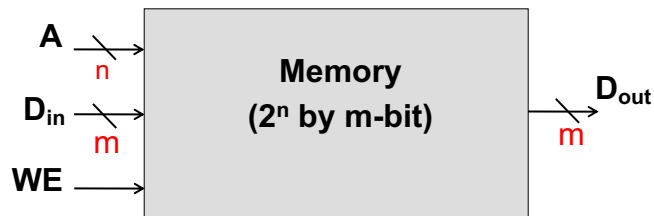
m bits

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Memory Interface

- ❖ There are two basic operations on a memory
 - Selecting one of the memory locations to **read** from
 - Selecting one of the memory locations to **write** to
- ❖ Interface signals
 - A: n -bit address lines to select/specify a location
 - D_{out} : Contents of selected location during read (m bits)
 - D_{in} : Value to be stored during write (m bits)
 - WE: If WE = 1 then write operation, WE = 0, read operation

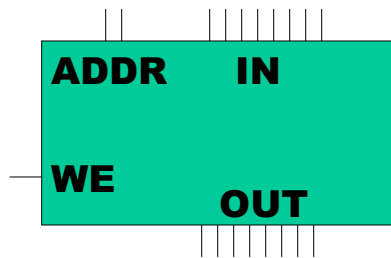


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Memory

- Looking from the outside, what do we need?



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Memory

A large number of addressable fixed size locations

- Address Space

n bits allow the addressing of 2^n memory locations.

- Example: 24 bits can address $2^{24} = 16,777,216$ locations (i.e. 16M locations).
- If each location holds 1 byte (= 8 bits) then the memory is 16MB.
- If each location holds one word (32 bits = 4 bytes) then it is 64 MB.

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Memory

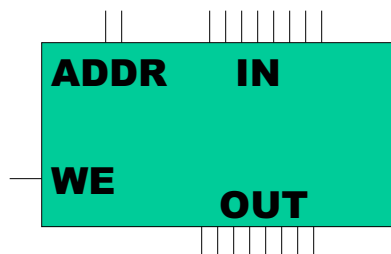
- Addressability
 - Computers are either **byte** or **word** addressable - i.e. each memory location holds either 8 bits (1 byte), or a full standard word for that computer (16 bits for the LC-3, more typically 32 bits, though now many machines use 64 bit words).
- Normally, a whole word is written and read at a time:
 - If the computer is **word** addressable, this is simply a single address location.
 - If the computer is **byte** addressable, and uses a multi-byte word, then the word address is conventionally either that of its most significant byte (**big endian** machines) or of its least significant byte (**little endian** machines).

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Memory

- READ operation: Given address A of N bits, fetch contents at that address
 - From 2^N locations we *select* one of them to be sent to the output
- WRITE: Given address A of N bits, write into exactly one of the 2^N locations.



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Devices to construct Memory ?

- To store a m-bit number use a "register" (m-bit D-latch)
 - To store 2^N of these m-bit numbers, use 2^N m-bit latches
- For READ: what is the device that can send one out of 2^N inputs (inputs are in the 2^N latches) ?
- For WRITE: what is the device that can enable exactly one Write Enable (WE) from the 2^N D-latches ?

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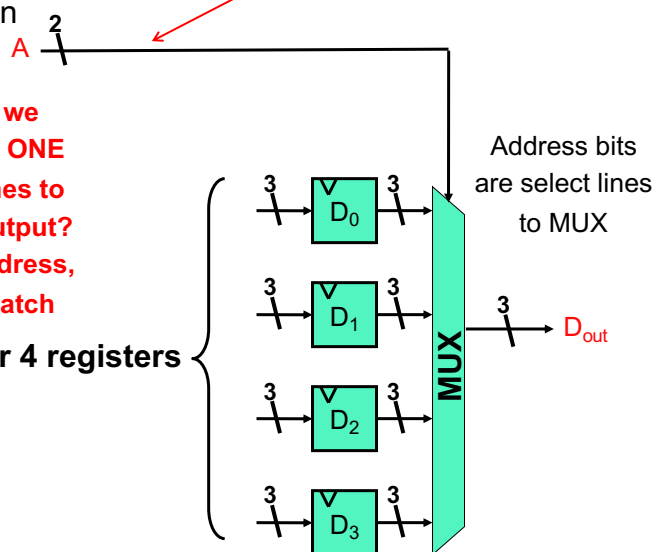
2^2 by 3-bit memory

- Read operation

But how do we select/enable ONE of the D-latches to send to the output?
Given 2 bit address,
Select ONE latch

2^2 or 4 registers

Selects "address" to read



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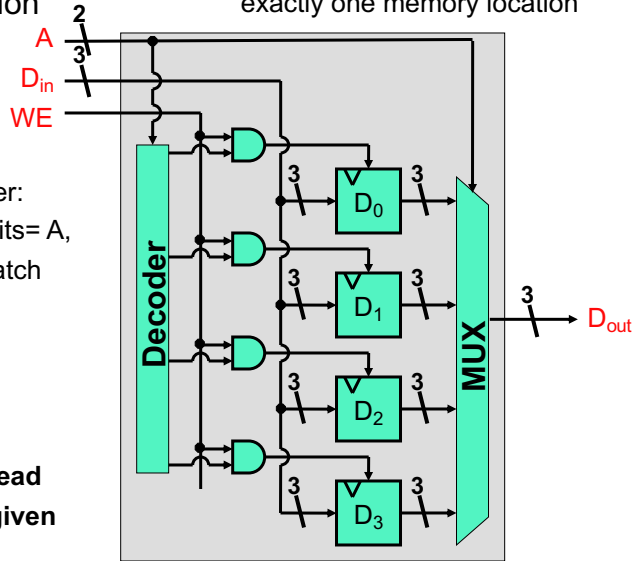
2² by 3-bit memory

- Write operation

How do we enable write into exactly one memory location

Use 2-4 Decoder:
Input address bits= A,
exactly one D-latch
has WE=1

Limitation:
You can only read
or write at any given
time



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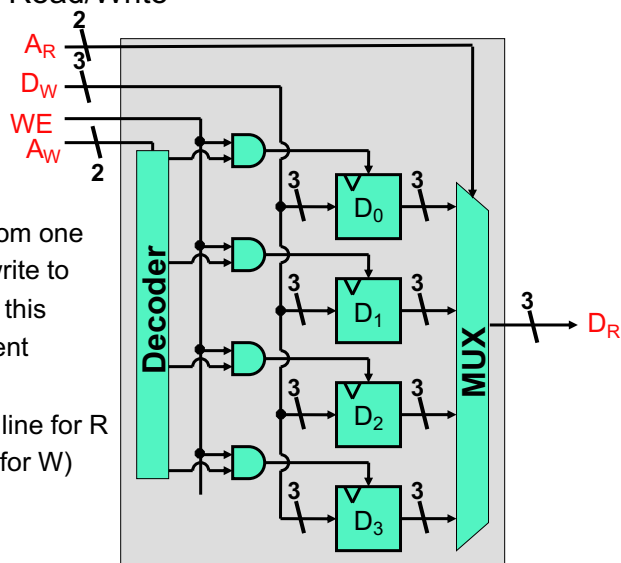
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2² by 3-bit memory - Multiple “Ports”

- Independent Read/Write

You can read from one
address and write to
another with this
arrangement

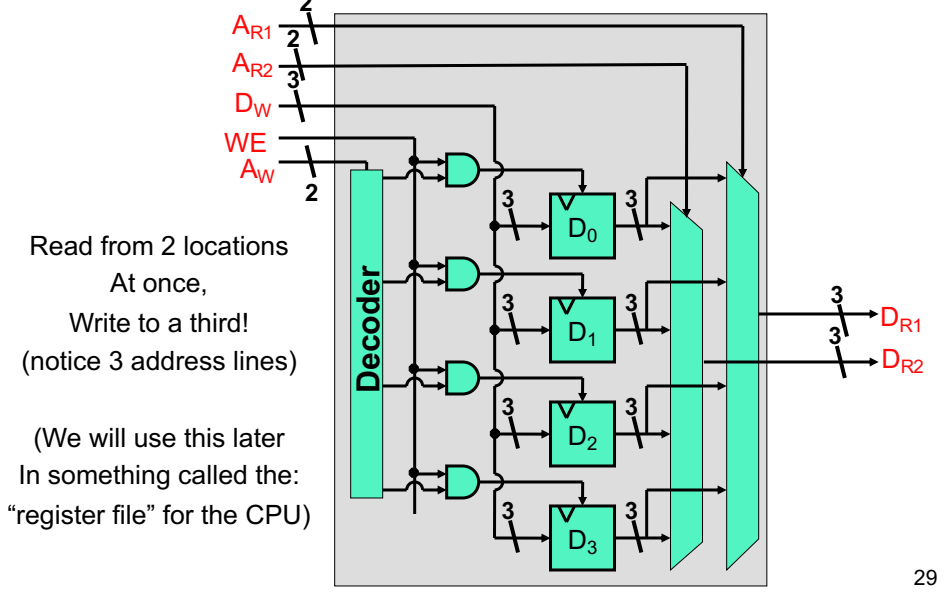
(notice 1 address line for R
1 address line for W)



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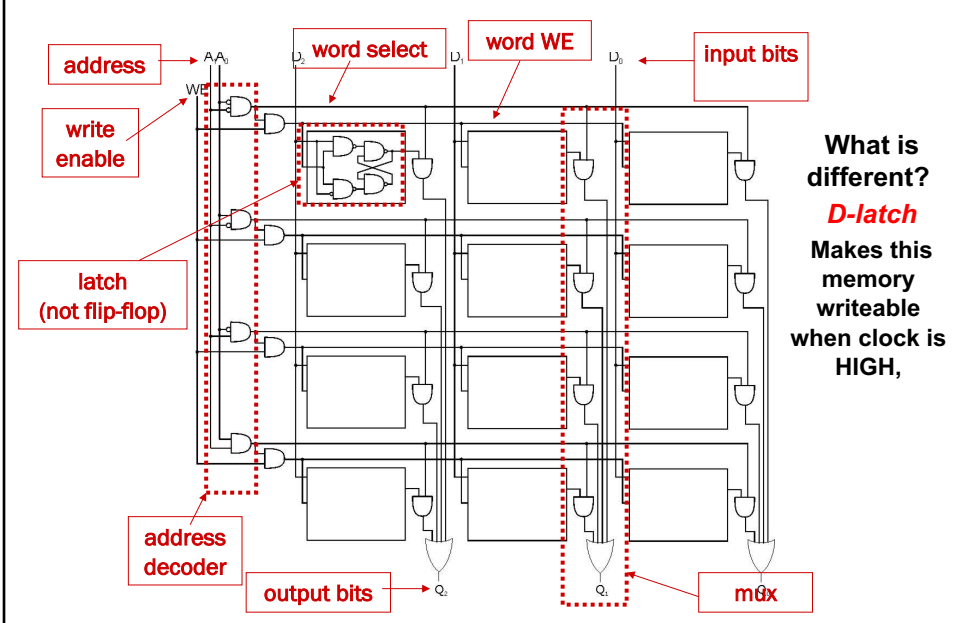
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2² by 3-bit memory - Multiple Read Ports



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Inside an Efficient 2² by 3-bit Memory - Single Port



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More Memory Details

- This is still not the way actual memory is implemented
 - Real memory: fewer transistors, denser, relies on analog properties
- But the logical structure is similar
 - Address decoder
 - Word select line, word write enable
 - Bit line
- Two basic kinds of **RAM** (Random Access Memory)
 - **Static RAM** (SRAM) - 6 transistors per bit
 - Fast, maintains data as long as power applied
 - **Dynamic RAM** (DRAM) - 1 transistor per bit
 - Denser but slower, relies on “capacitance” to store data, needs constant “refreshing” of data to hold charge on capacitor

Also, non-volatile memories: ROM, PROM, flash, ...

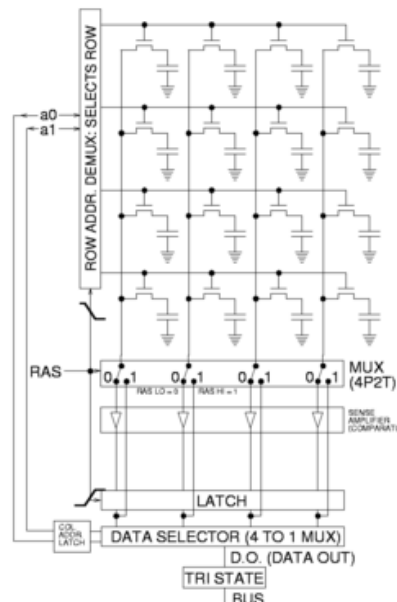
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Dynamic RAM

- Information stored as charge on capacitors.
- Capacitors leak so values have to be ‘refreshed’ continually
- As memory chips get larger, access times tend to increase. The processor spends more time waiting for data.

➤ This is a **major** issue limiting computer systems performance



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Speed mismatch: Example

- Intel Core i5 – Processor
 - Clock rates approx 2.5GHz, Clock period approx 0.4 ns
- DDR2-667 PC2-5300 SO-DIMM – 2 GB Memory
 - Can deliver at most 1 64-bit word every 1.5 ns
- Mismatch between processor speed and memory speed

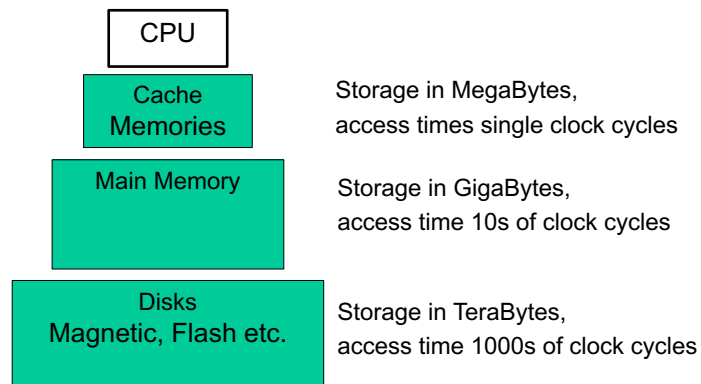


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Memory Hierarchy

- Modern computers try to mitigate memory delays by exploiting locality of reference through caches.
- Smaller, faster memory stores are placed closer to the CPU and bulk transfers from slower memory are used



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Memory Hierarchy

- Will return to this at the end of the course....!

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Are we ready to design sequential circuits and finite state machines ?

- Is something missing ?
- When do states change in a machine ?
- Do we let states change at arbitrary times ?
- What do you think happens in a computer ?

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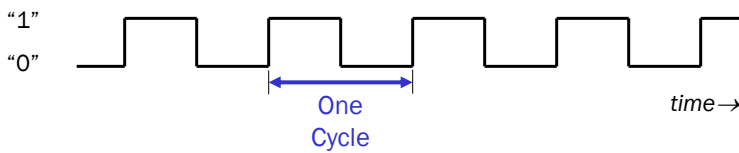
Clocked Flip-Flops/Circuits

- Subsystem in a computer consists of a large number of combinational and sequential devices
 - Each sequential device is like latch which is in one of two states
 - As machine executes its cycle, the states of all sequential devices change with time
- To control large collection of devices in an orderly (synchronized) fashion, machine maintains a **clock**
 - Requires all devices to change their states at the same time
 - Clock generates sequence of pulses
- Much easier to design, debug, implement, and test
- How do we change latches so that they allow change in state synchronized with the clock ?
- Sequential logic circuits require a means by which events can be sequenced.....clock!

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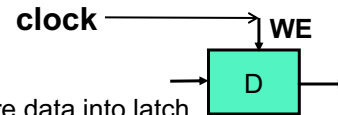
Introducing - The Clock!

- A **clock** controls when stored values are “updated”
 - Electrical waveform – sends pulses through a circuit
 - Changes values at a periodic rate
- 
- The clock will act as the ‘heartbeat’ of our system
 - The number of cycles per second is the **clock frequency** measured in cycles per second or Hertz (Hz)
 - The **clock period** refers to the duration of one clock cycle. The period and frequency are inversely related.
 - Typical clock frequency: $2.5\text{GHz} = 2.5 \times 10^9 \text{ Hz}$
 - So corresponding clock period = $1/(2.5 \times 10^9) = .4 \times 10^{-9} \text{ sec}$
 - That would be: 0.4 nanoseconds

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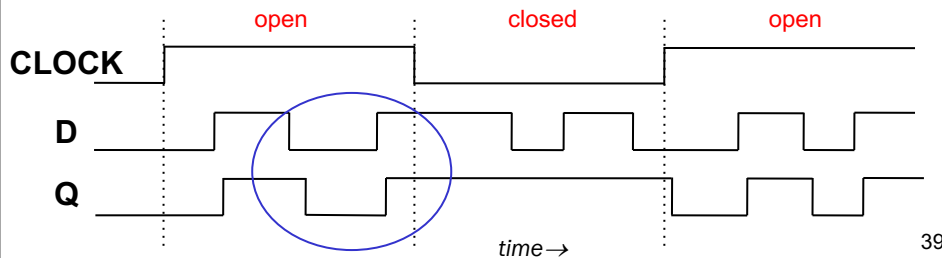
Attaching Clock to D-Latch



- Attach CLOCK to the WE on D-Latch
- We create “windows” of time that we can store data into latch
 - When the CLOCK is “HIGH” – D-latch is open
 - When the CLOCK is “LOW” – D-latch is closed
- D=Q while CLOCK is High
- We have to prepare what we wish to store, right before latch closes

Oops...changes in input while clock=High cascade to output Q

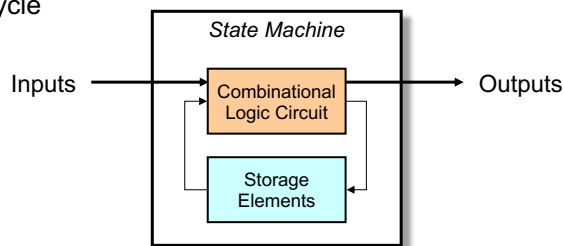
- Q can change multiple times during clock cycle: not synchronized with clock



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Why Clocked D-latches may not work

- Sequential circuit: the next state (and output) depend on values of current state and input
- If input to D-latch changes while clock is High, then next state (and output) can change during this single clock cycle
 - state we want to store as next state could be overwritten
- we want to force changes (in state and input) to be synchronized to the clock
 - allow input/states to be read and output/next state to change ONCE per clock cycle



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Enter the Flip Flop....

- Flip flops are edge triggered devices
 - They capture the input and change state when the clock changes level
 - Positive edge triggered: when clock goes from 0 to 1
 - Negative edge triggered: when clock goes from 1 to 0
- We refer to the D flip flop as an **edge-triggered** device.
 - D=Q ONLY when WE changes from 0 to 1
- This differs from D latch, which is: **level-triggered**
 - D=Q anytime WE equals 1

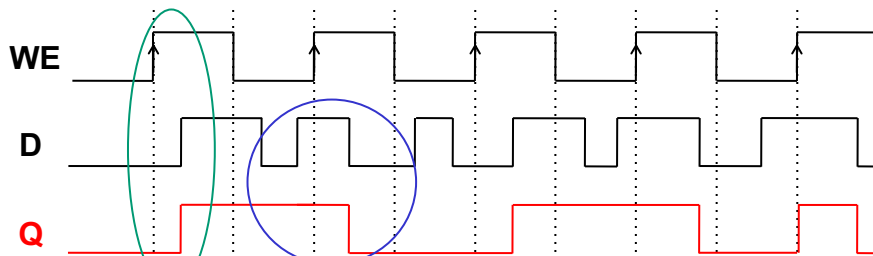
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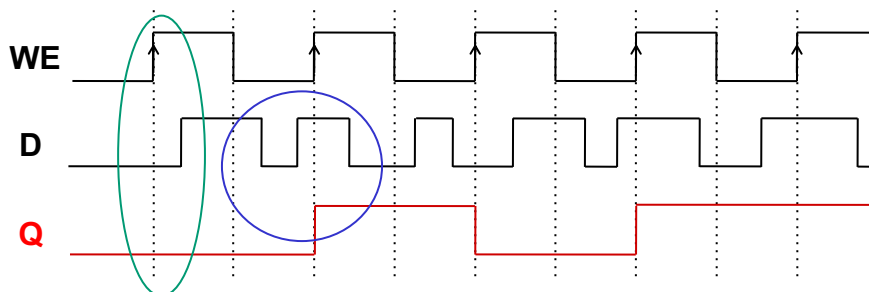
D Flip-Flop vs. D-Latch – Timing Diagrams

Observe difference in how output Q synchronizes with clock edge in DFF

Timing Diagram for D-Latch:



Timing Diagram for DFF:



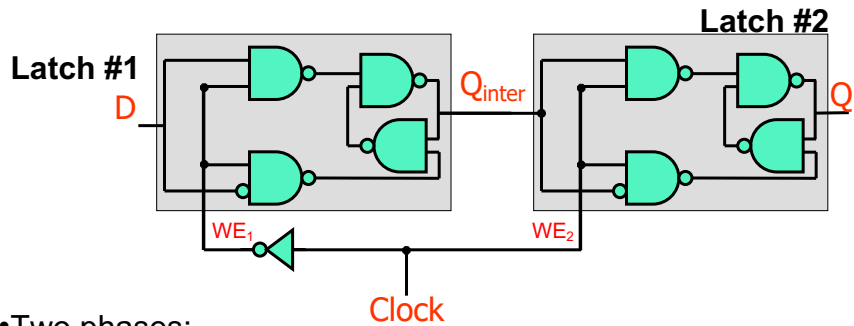
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D Flip-Flop

• D Flip-Flop is a pair of D latches

- Isolates *next* state from *current* state
- Output of DFF read when clock is high, Next state stored end of cycle



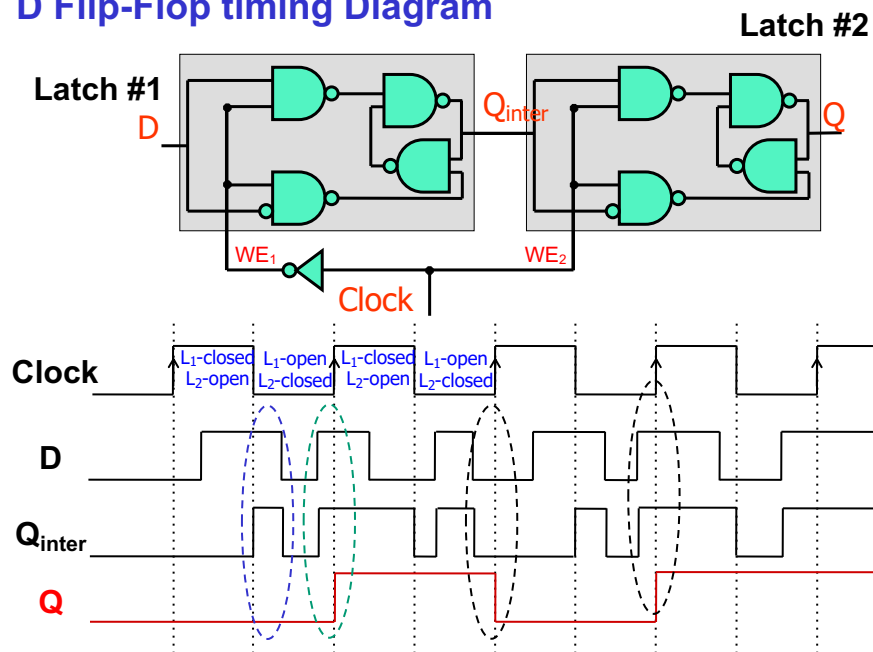
• Two phases:

- Clock = 1: $WE_1 = 0$: Latch #1 closed, $WE_2 = 1$: Latch #2 open
- Clock = 0: $WE_1 = 1$: Latch #1 open, $WE_2 = 0$: Latch #2 closed

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D Flip-Flop timing Diagram





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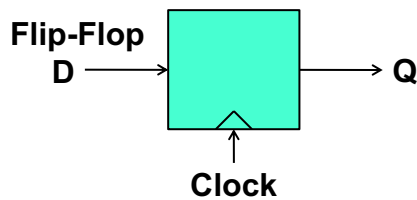
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D Flip-Flop

- We can think of the D Flip-Flop as a 1 bit storage container with an input, D, and an output, Q and connected to a clock input
- A set of D flip-flops can be grouped together with common Clock and WE inputs to form a **register** -- replace D-latch with D flip flop

Truth table for DFF:
X = don't care (i.e, 0 or 1)

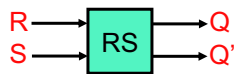
| D | Clk | Q |
|---|---|--------|
| 0 | 0  | 0 |
| 1 | 0  | 1 |
| X | 0 | Last Q |
| X | 1 | Last Q |



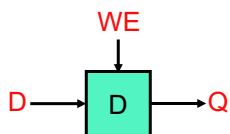
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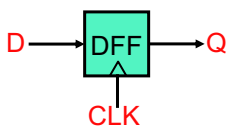
Our basic "Storage" Devices



RS Latch – Stores 1 Bit, Level-Triggered
-1 "forbidden" input: S=0, R=0
-Holds Data when RS=11



D-Latch – Stores 1 Bit, Level-Triggered
-No "forbidden" inputs (fixes RS Latch)
-D=Q when WE=1
-Holds Data when WE=0



D-Flip-Flop – Stores 1 Bit, Edge-Triggered
-No "forbidden" inputs
-D=Q when WE (CLK) transitions from 0 to 1
-Holds Data for WE=1 or WE=0
-Except when WE transitions from 0 to 1

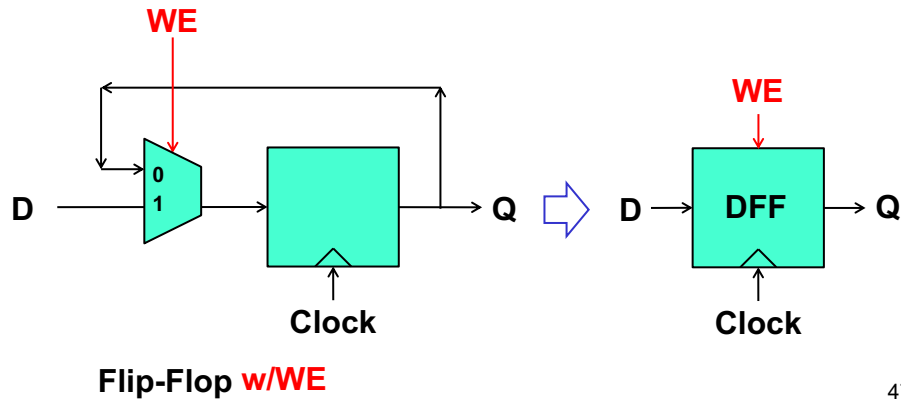
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One more tweak...

D Flip-Flop with Additional Write Enable

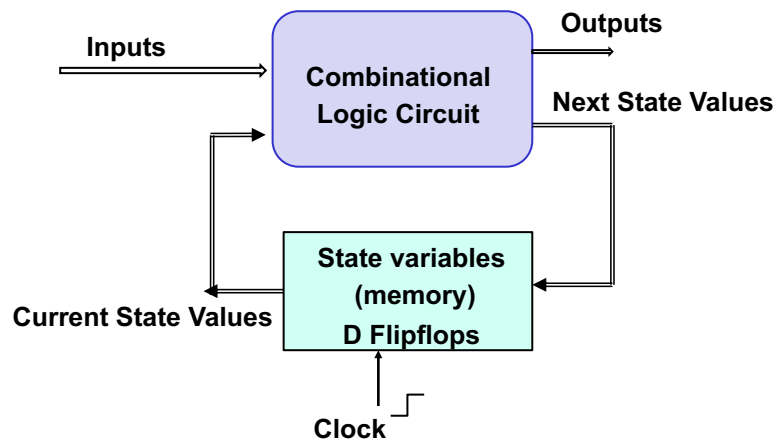
- From previous slides, we attached clock to WE of the D-flip-flop
- Now, we add another WE line to the flip flop
 - Just holds onto data already stored in DFF
- Give it the ability to “ignore” the clock!



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Schematic of a Sequential Circuit



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Next: Design methodology for sequential logic circuits .. Finite State Machines

- We have storage devices and method for synchronization
 - Flip flops and Clock
- How do we design a clocked circuit to solve our problems ?
 - Example: Is there a methodology behind designing a circuit to implement a Counter ?
- Provide procedure for designing Sequential circuits (to implement Finite State Machine)
 - Storage Device to store state: D flip flop
 - Logic to implement next state: combinational gates/devices
 - How to derive the logic: truth tables

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States in a FSM

- The concept of state
 - the **state** of a system is a “**snapshot**” of all relevant elements at a moment in time.
 - a given system will often have only a **finite number of possible states**.
 - For many systems, we can define the rule which determines under what conditions a system can **move from one state to another**.
 - So when do they change states ?
 - Synchronized to clock (edge triggered)
- Determining the ‘states’
 - Problem statement determines what information needs to be stored (state is a summary)
 - How many states does the machine need ?

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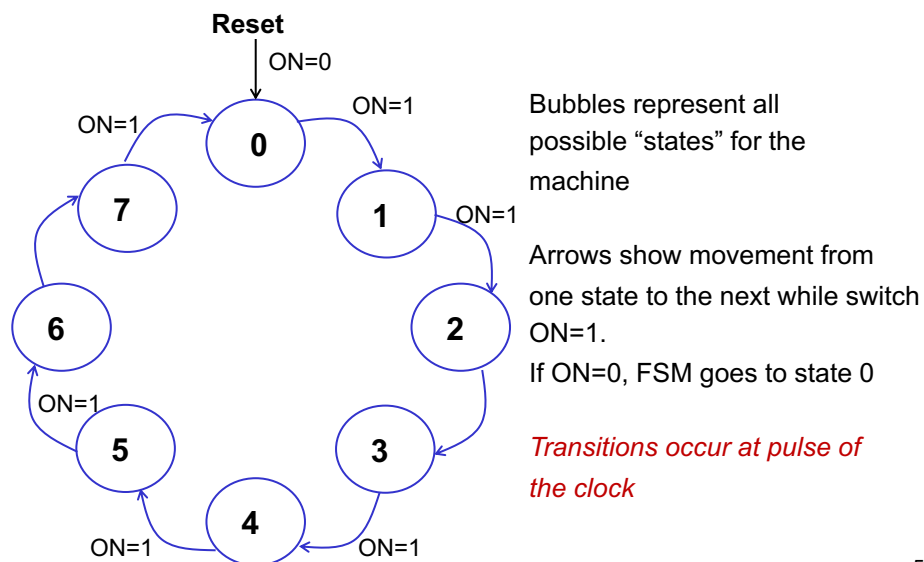
Designing and implementing a FSM

1. First draw the state diagram
 - Encode each state in binary using N bits
 - These N bits correspond to N “state variables” that need to be stored. Call them $S_{N-1} S_{N-2} \dots S_1 S_0$
 - State diagram will show transitions from state to state based on value of inputs
2. Next, derive the truth table (from state diagram)
 - “inputs” in truth table are N current state variables and the inputs
 - “outputs” are the values of the state variables in the next state and the output at each state -- common notation is S' but confusion with complement operator, so let's use S^*
3. From truth table, derive combinational circuit (boolean function) for each of the next state values
 - State variables are stored in your N storage elements

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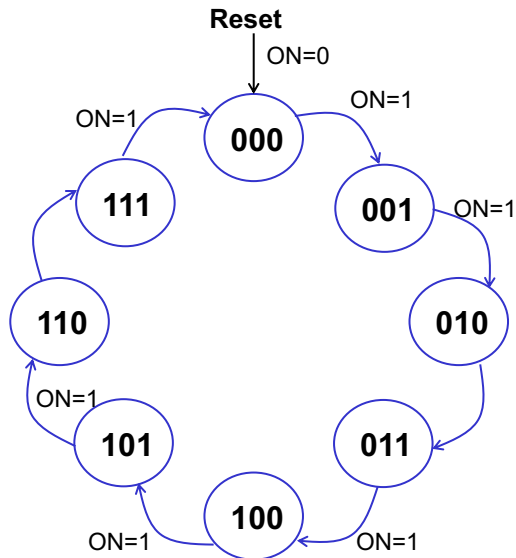
Finite State Machine Representation of Counter: Counter to count from 0 to 7 (while switch is ON)



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Labelling States in Binary



We have 8 states, therefore we need $\log_2 8 = 3$ bits to encode states in binary:
from 000 to 111

Each bit is called a state variable & Stored in one D flip flop
we need 3 Flip flops to store 3 bits $S_2 S_1 S_0$

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Storage

- Each D flip flop stores one state bit.
- The number of storage elements (flip-flops) needed is determined by the number of states (and the representation of each state).
 - Each bit can be 0 or 1 = 2 states
 - N bits can represent 2^N states
- Example: If FSM has 12 states, then circuit needs $\log_2 12 = 4$ storage elements (i.e., flip flops).
 - Fewer the states, less hardware needed
 - *Concept of Minimization of States for a given FSM...*
 - ...in Foundations (where else!!)

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Designing and implementing a FSM

1. First draw the state diagram
 - Encode each state in binary using N bits
 - These N bits correspond to N “state variables” that need to be stored. Call them $S_{N-1} S_{N-2} \dots S_1 S_0$
 - State diagram will show transitions from state to state based on value of inputs
2. Next, derive the truth table (from state diagram)
 - “inputs” in truth table are N current state variables and the inputs
 - “outputs” are the values of the state variables in the next state S^* and the output at each state (common notation is S' but confusion with complement operator, so let's use S^*)
3. From truth table, derive combinational circuit (boolean function) for each of the next state values
 - State variables are stored in your N storage elements

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Truth Table Representation of Counter

| Input | Present State | | | Next State | | |
|-------|---------------|----------|----------|--------------|--------------|--------------|
| | $S_2(t)$ | $S_1(t)$ | $S_0(t)$ | $S_2^*(t+1)$ | $S_1^*(t+1)$ | $S_0^*(t+1)$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | X | X | X | 0 | 0 | 0 |

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This part is no different from combinational circuit design!

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Boolean functions for values of next state

- For each state variable, derive function that determines next state for that variable:
- Note: $S_2(t+1)$ – value of state variable at time (t+1) is denoted as S_2^*
- Current states: $S_2 S_1 S_0$
- Derivation here is only for case when $On=1$

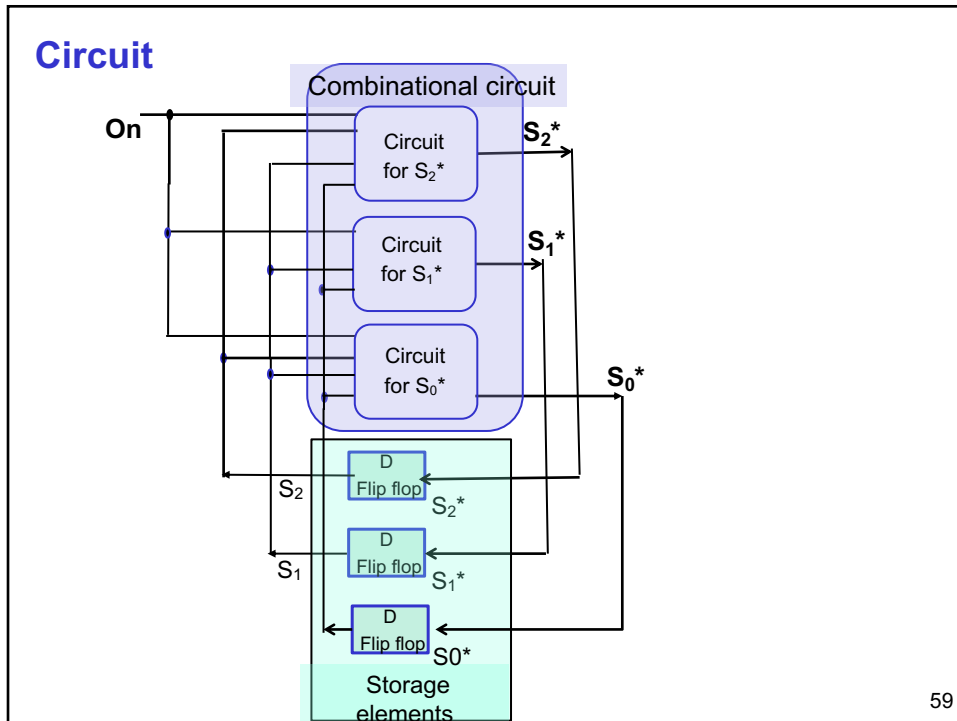
- $S_2^* = On.(S_2' S_1 S_0 + S_2 S_1' S_0' + S_2 S_1' S_0 + S_2 S_1 S_0')$

- $S_1^* = On.(S_2' S_1' S_0 + S_2' S_1 S_0' + S_2 S_1' S_0 + S_2 S_1 S_0')$

- $S_0^* = On.(S_2' S_1' S_0' + S_2' S_1 S_0' + S_2 S_1' S_0' + S_2 S_1 S_0')$

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Summary: Designing and implementing a FSM

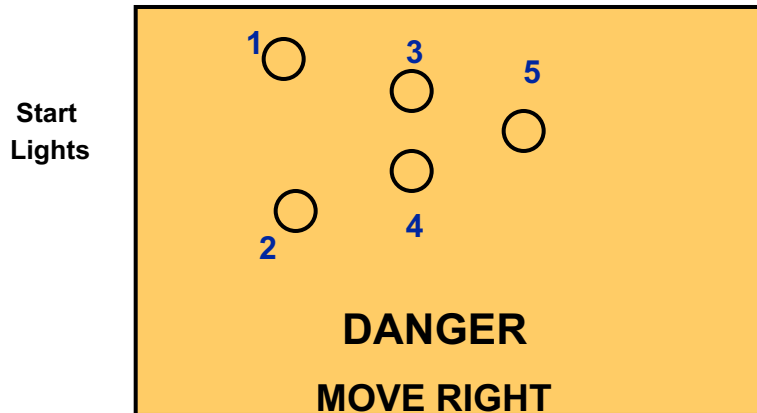
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Example 2: Blinking Traffic Sign: Start all lights off – lights bulbs 1,2,3,4,5 off

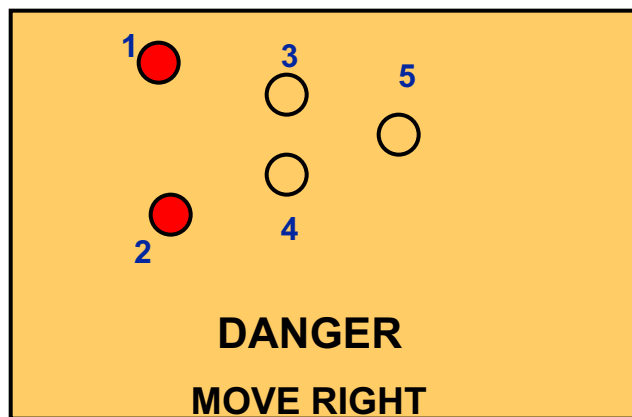
- (from textbook) Design control circuitry for a blinking traffic sign (to show “move right” message)



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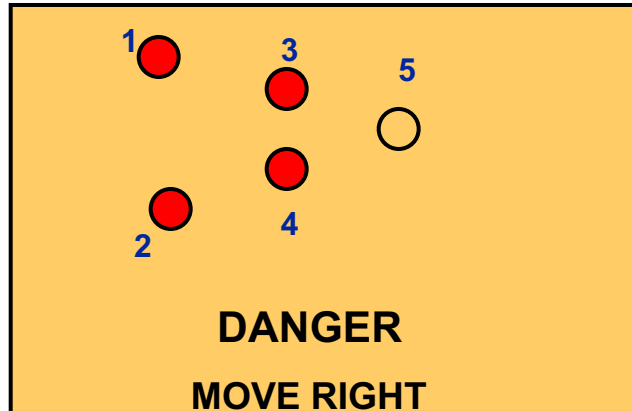
Step2: switch ‘on’ 2 lights – bulbs 1,2



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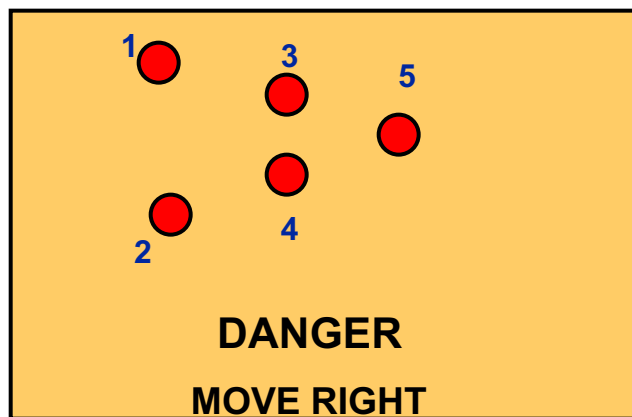
Step 3: Switch 'on' 4 lights – bulbs 1,2,3,4



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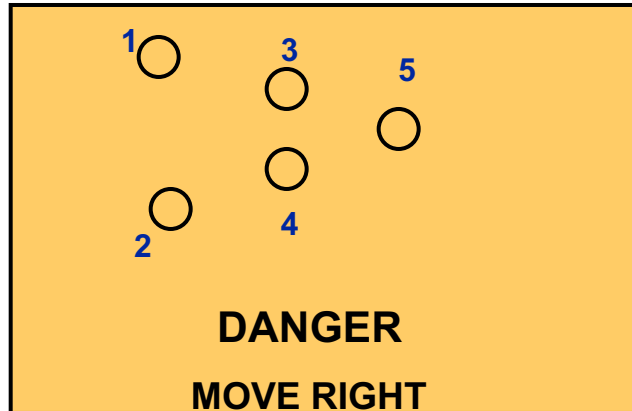
Step 4: switch 'on' 5 lights – 1,2,3,4,5



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**Step 5- switch all 'off' – lights bulbs 1,2,3,4,5 off
And repeat the cycle**



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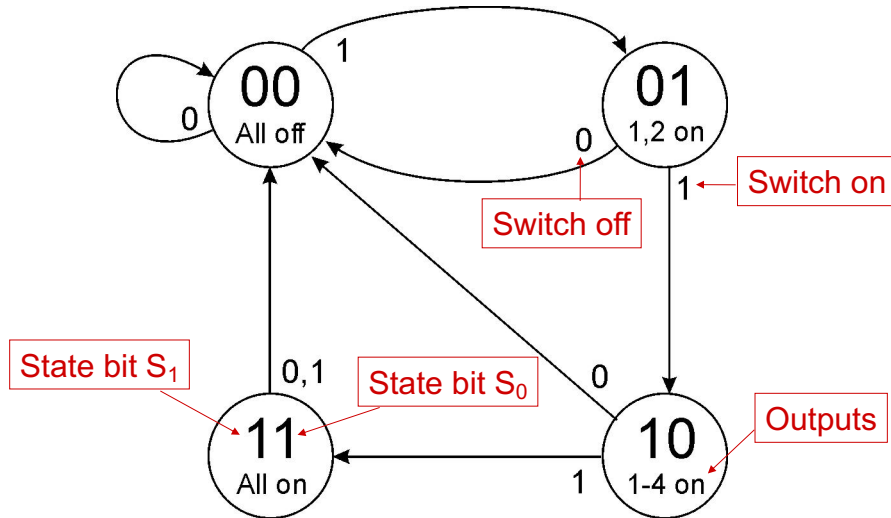
Example: Traffic Sign

- A blinking traffic sign: How many lights/lightbulbs = 5
- How many states ?
- 4 states
 - No lights on
 - 1 & 2 on
 - 1, 2, 3, & 4 on
 - 1, 2, 3, 4, & 5 on
 - (repeat as long as switch is turned on)
- How many bits to represent the 4 states
- S_1S_0
 - With S_1S_0 values: 00, 01, 10, 11
- How many 'outputs' (to control the 5 lights) = 5 ?
- If the sign is switched off then all lights turn off

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Traffic Sign State Diagram



Transition on each clock cycle.

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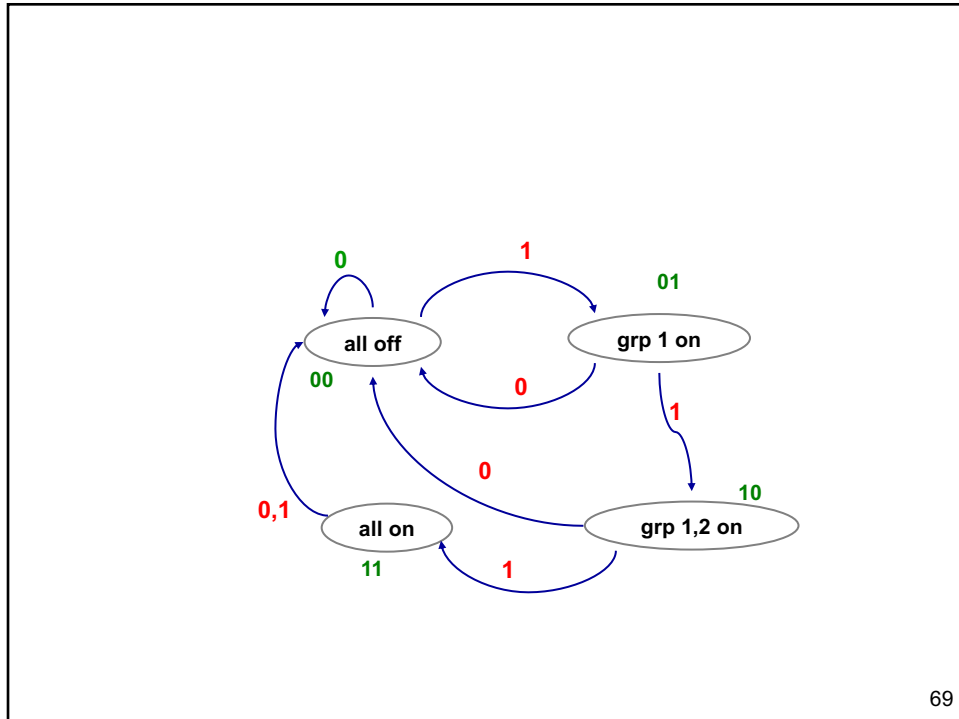
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Outputs

- Note we really have 3 groups of lights to be controlled = 3 control lines X,Y,Z
 - Group 1: Lights 1 and 2; controlled by Z
 - If Z=1 then Group 1 lights (1 and 2) are switched on
 - Group 2: lights 3 & 4; controlled by Y
 - Group 3: Light 5; controlled by X
- In this example, we associate each state with an output
 - Depending on the current state, we switch on specific groups of lights

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- When is group 1 on?
 - in states 01, 10 and 11 - but only when the switch IN is on!
- Logic expressions for X,Y,Z
 - Depends on S_0 and S_1 and Input is on
 - If Input is off then X,Y,Z are all 0
- can you come up with a logic expression for next state values of S_0 and S_1 ?
 - Depends on current values of S_0 and S_1 and Input is on
 - Input off then both bits are set to 0 since next state is 00
 - Next state value of S_0 denoted $S'_0 = 1$ if current state is 00 or current state 10 and $In=1$
- When do we switch to the next state?
 - the two bits of $S[1:0]$ are updated at every clock cycle
 - we have to make sure that the new state does not propagate to the combinational circuit input until the next clock cycle.

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Traffic Sign Truth Tables

Outputs
(depend only on state: S_1S_0)

| S_1 | S_0 | Z | Y | X |
|-------|-------|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

Lights 1 and 2
Lights 3 and 4
Light 5

Next State: $S_1'S_0'$
(depend on state and input)

| In | S_1 | S_0 | S_1' | S_0' |
|----|-------|-------|--------|--------|
| 0 | X | X | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |

Switch

Whenever In=0, next state is 00.

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Boolean functions for light control bits

- from truth table, consider all rows where outputs =1

- $Z = ((\text{NOT } S_1)S_0 + S_1(\text{NOT } S_0) + S_1S_0).In$

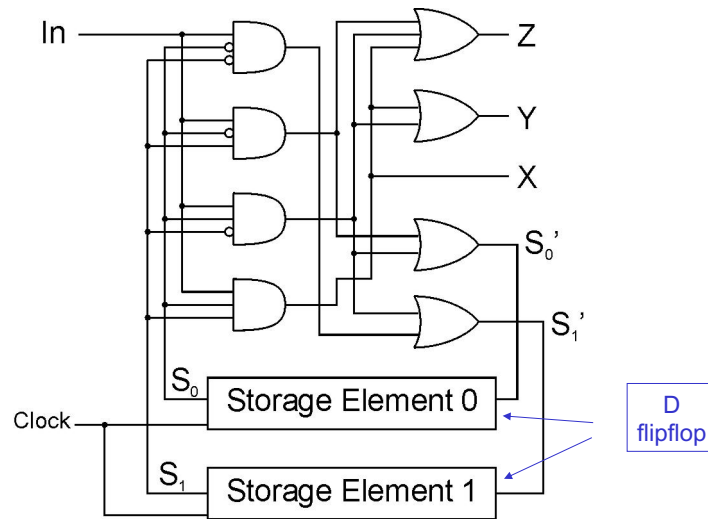
- $Y = (S_1S_0 + S_1(\text{NOT } S_0)).In$

- $X = (S_1S_0).In$

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Traffic Sign Logic



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Exercise: Design a sequential circuit

- Counter that counts from 0 to 6
 - Resets to 0 after 6
 - Keeps going as long as "input" = 1
 - Ignore this for now
- Finite state diagram
- Truth table
- Logic functions for next state

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Summary of Digital Logic

- Combinational logic
 - Basic gates
 - Combinational devices
- Sequential logic
 - Storage element... Flip flop
 - Theory behind design of finite state machines
 - They act like controllers of a circuit
- Sequential logic devices
 - Memory, ROM, RAM, Registers

So, what is the purpose of all this ?.....

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The Agenda

- Take the elements that we have encountered so far
 - Combinational Elements
 - Gates, Adders, Muxes, decoders
 - Storage Elements
 - Flip flops, registers, memories
- And use them to build a circuit that can perform a **sequence** of arithmetic operations.
 - In essence, we will build a very simple CPU

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From Logic to Processor Design

•Combinational Logic

- Decoders -- convert instructions into control signals
- Multiplexers -- select inputs and outputs
- ALU (Arithmetic and Logic Unit) -- operations on data

•Sequential Logic

- State machine -- coordinate control signals and data movement
- Registers and latches -- storage elements

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Next: Putting it all together

- The goal:
 - Turn a theoretical device - Turing's Universal Computational Machine - into an actual computer ...
 - ... interacting with data and instructions from the outside world, and producing output data.
- Smart building blocks:
 - We have at our disposal a powerful collection of combinational and sequential logic devices.
- Now we need a master plan ...
 - We need to start with defining the model of a computer architecture
 - **Von Neuman model**

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